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BANKS' REGULATORY CAPITAL REQUIREMENT: PRICING THE CREDIT RISK OF SHORT-TERM LOAN COMMITMENTS

BY

J. – P. CHATEAU and D. DUFRESNE

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J. - P. CHATEAU [a] and D. DUFRESNE [b]
Chat@zoo.net Dufresne@dms.umontreal.ca

ABSTRACT

According to the accounting-based valuation procedure mandated by the Bank for International Settlements (the BIS), banks' off-balance-sheet short-term loan commitments do not carry any credit risk, and hence, do not require any capital charge. By way of contrast, we propose here that: 1) the actual as well as potential credit risk of each bank instrument be linked more accurately to the regulatory capital charge, and 2) when applying this to short-term commitments, their risk-weighted balance (used in computing the BIS solvency ratio) be based on their derivative-based fair value. To this effect, we first note that the fixed forward markup of floating-rate commitments gives rise to the credit-line marked-to-market value, x, and subsequently, to an embedded commitment put option, P. The latter is valued in a two-factor model of x and the default-free but stochastic short-term interest rate, r. Once computed, the put value is combined with the line fees and a conditional exercise-cum-takedown proportion to determine the commitment net value and the bank exposure to commitment credit risk. The main pattern emerging from the simulation experiments is that commitment put values constitute credit-risk liabilities. It then results that put values and the exercise-cum-takedown proportion can be used to compute the positive risk-weighted balance, which links short-term commitments to the regulatory capital charge.

Keywords: commitment put value, commitment net value, bank's exposure to commitment credit risk, and BIS regulatory capital requirement.
Journal of Economic Literature: classification: G13 and G21

[a] Rouen Graduate School of Management, France; [b] Department of Mathematics and Statistics, University of Montreal, Canada. This article is based on a research report (Chateau and Dufresne, [1999]) funded by a grant from the Committee on Knowledge Extension Research (CKER) of the Society of Actuaries. We thank participants at the International Conference in Finance in Aix-en-Provence, and research seminars at UQAM, HEC-Montreal, the University of Montreal and the University of Cergy-Pontoise for helpful comments and discussions. Address correspondence regarding file 2facmod.03 dated January 2002, to J–P Chateau, ESC-Rouen, Blvd A. Siegfried --PB 188, 76825 Mont-Saint-Aignan Cedex, France. Phone/Fax: +33 2 32 82 57 29.

BANKS' REGULATORY CAPITAL REQUIREMENT:
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1. INTRODUCTION

The accounting-based capital requirement mandated by the Bank for International Settlements (the BIS) allows for some spectacular cases of regulatory arbitrage. The most blatant one is the banks' preference for short over longer-term loan commitments or more precisely, the preference for those with an initial term to maturity up to one year over those with an original term to maturity over one year. The banks' reasoning is simple: 364-day credit commitments are not subject to any capital constraint whereas the risk-weighted, positive balance of longer-term commitments is used to compute the banks' regulatory capital requirement. Yet, this artificial dichotomy in commitment maturity could be eliminated if the commitment potential risk is linked accurately to the regulatory capital charged. This observation raises two questions: 1) What is the fair value of credit line (CL) commitments? And 2) Do banks incur any (even notional) liability when offering loan commitments, and if so, how is their credit-risk exposure computed?

Thakor *et al.* (1981) have shown that a CL commitment can be viewed as a put option sold by the bank to its borrowers. When the commitment interest rate is lower than that on an equivalent spot loan, the borrower receives the line face value but is only indebted for its lower marked-to-market value (henceforth to be referred to as the **indebtedness value)**. This borrower's claim on the lending bank constitutes an embedded, yet valuable, commitment put option. The aggregate face value of still unused commitments is reported as an off-balance-sheet entry to the bank's consolidated balance sheet and is subject to regular (monthly, quarterly, and by law, annual) audits. As Merton (1977) has argued for related loan guarantees, the time remaining to commitment maturity can be interpreted as the length of time until the next audit of these off-balance sheet contracts. In that case, the boundary condition of the commitment put is Max $(L - x_T, 0)$, where L denotes the CL par value and x_T its indebtedness value at the annual audit date, T. The value of the ***European*** commitment put thus captures the bank's notional liability for carrying off-balance sheet commitments at the audit date. In this research, we examine the most prevalent type of CL commitments, those with a floating-rate formula devised as "stochastic index cost of funds plus a fixed forward markup". And amongst those, we concentrate on the class of prime-rate[1] commitments with an original term to maturity less than one year. These short-term commitments are utilised for general corporate purposes or to finance working capital, trade and commerce.

In recent years, several researchers have derived alternative formulas for valuing bank credit line commitments. Thakor *et al*. (1981), and Ho and Saunders (1983) derived option-like expressions for fixed-rate CL commitments, Thakor (1982), Chateau (1990), and Chateau and Dufresne (2001) obtained valuation formulas for variable-rate line commitments and Hawkins (1982) priced revolving credit lines. All chose however to retain the assumption that the discount factor used in valuing the commitment put option is a constant risk-adjusted interest rate. In actuality, the short-term interest rate is stochastic and variable and may or may not be correlated with the fixed forward markup in the commitment floating-rate formula.

Granted this observation, we begin the formal analysis by introducing dynamics for (i) the indebtedness value and (ii) the short-term interest rate. The latter presence is due to the existence of a discount factor in the commitment valuation formula. This short rate and its dynamics should not be confused with (and are not a proxy for) the rate on certificates of deposits (CDs), the "exogenous-cost-of-funds" component of the commitment floating-rate formula. As this cost is borne by the borrower, it does not enter credit risk pricing. Next, given short-rate and indebtedness-value diffusion processes, we value analytically the European commitment put in three stages. In stage I, a default-free discount bond is priced as in Hull and White (1990). It is next used, in stage II, to obtain the closed-form expression for the European commitment put when the default-free, short-term interest rate is stochastic. And in stage III, commitment fees and put value are combined to determine the commitment net value, which in turn is used to compute the bank's exposure to commitment credit risk. The novelty for both concepts is that an exercise-indicator function is combined with a conditional takedown proportion that accounts for line partial funding. The two-

[1] For non-prime commitments, the rate is rescaled to adjust for prime rate add-ons or prime rate discounts; as for the magnitude of such spreads over the floating prime rate, consult Angbazo *et al.* (1998) or Elsas *et al.* (1998).

factor commitment put model is then compared to the corresponding Black and Scholes (B-S) one-factor put formula. This comparison brings to the fore the following differences. The constant risk-free discount rate used in B-S is replaced by a variable default-free discount factor in the present model. The one-factor Gaussian term structure also affects the commitment put volatility differently. In the B-S formula, the put volatility reduces to that of the indebtedness value. In the proposed model, three additional terms capture: (i) the volatility of the short-term interest rate, and (ii) the correlation between the fluctuations in this rate and the changes in the indebtedness value.

As commitment puts are but notional values of embedded credit-risk derivatives, we next turn to simulation experiments to support the above theoretical considerations. The purpose of these simulations is threefold. First, to uncover value differences between the proposed model and the B-S formula when: (i) the correlation between the short rate, r, and the indebtedness value, x, varies in the interval [-1 to +1], and (ii) when the short-rate volatility fluctuates in the (empirically relevant) range of [2% p.a. to 10% p.a.]. Second, to detect whether the proposed put value is more sensitive to the (x - r) correlation than to the short-rate volatility. And third, to determine on the basis of computed put estimates, reasonable line fees and an exercise-cum-takedown proportion, the commitment net value and the bank's exposure to commitment credit risk. Finally, in the light of the numerical values, we examine whether the risk-adjusted balance of short-term commitments should be determined by the BIS accounting-based procedure or the option-based valuation proposed here.

The rest of the paper is organized as follows. In Section 2 we value analytically the European commitment put, and determine the commitment net value and the bank's commitment risk exposure. Simulation results are presented in Section 3 and used next in Section 4 to quantify the link between short commitments and bank capital charge. The paper concludes in Section 5 with a short summary.

2. VALUATION OF FLOATING-RATE CREDIT COMMITMENTS

2.1 Problem statement

The important features of a fixed-markup commitment are stylized in the decision chart below. Consider a bank that writes at date 0 an off-balance-sheet commitment contract for a credit line with

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DECISION CHART OF A BANK CREDIT COMMITMENT WITH A FIXED FORWARD MARKUP. The terms of reference assume the absence of compensating deposit balances and bank reserve requirement[2].

a) Initial situation at t=0: commitment contractual terms.

One-year commitment period — \$100 one-year CL with a 1.5%-p.a. fixed markup ($\overline{m}_0$)

|---------------------------------+------------------------------|

t=0 — T — T_1

f^u_0 = 25¢: upfront commitment fee — f^e_T = 25¢: exercise-contingent usage fee

b) Situation at the valuation date s: T - s = 6 months is the remaining life of contract or Merton's (1977) time left to the next audit.

Exercise creates an on-balance-sheet loan. The bank is long a vulnerable repayment call

×------------------------------|

6 months ↑□ — T_1

|----------------x---------------+

t=0 — s — T ↓□

Valuation date — ×: Commitment expires as the \$100 credit line is left totally undrawn

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the following features: (i) the commitment period, [0, T], is one year; (ii) the CL face value, L, is

[2] American compensating deposit balances (Hawkins 1982) constitute a disguised cost to be treated here as a scaling problem. As for the bank reserve requirement, it was abolished in the early 1990s in Canada. There also exist minor annual fee and cancellation fee, in the order each

standardised at $100; (iii) loan duration, $[T, T_1]$, is one year from date T if the credit line is drawn down, and (iv) the commitment floating prime-rate formula is $[c_T + \overline{m}_0]$. Its first component, c_T, is the bank's **stochastic** cost of funds, with the rate on certificates of deposits (CDs) being generally used as exogenous index. The other component, $\overline{m}_0$, is the **fixed forward markup** that is determined when the commitment contract is written at date t=0. For instance, the $100 one-year CL has a time-0 (time-T) prime rate of 5.5% p.a. (6% p.a.) made up of a 4.0%-p.a. (4.5%-p.a.) stochastic cost of funds plus a fixed forward markup of 1.5% p.a. in both cases. This fixed markup signals to the market the creditworthiness of prime-rate borrowers at the time of commitment writing; as it only hedges credit risk, the corporate borrower either bears the funding risk, c, or takes an offsetting position in some interest-rate futures contract. Thakor and Udell (1987) provide the economic rationale for the bank's optimal deployment of commitment and usage fees in commitment pricing. In their competitive equilibrium model[3], the screening device resolves the bank-borrower asymmetries of information and the presence of adverse selection gives rise to split fees at the commitment end-dates. The upfront commitment fee guarantees the credit availability to the borrower while the end-date usage fee depends
on the borrower's private probabilities of future line utilisation[4]. In part (a) of the decision chart, the sorting variables are as follows: a commitment fee of 1/4 of 1% p.a., f^u_0, or here 25 cents per $100 of line face value, and an identical but exercise-contingent usage fee, f^e_T [5].

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of 3 to 5 basis points per annum (0.03 to 0.05 % p.a.).

[3] The formal exposition of Thakor and Udell's reactive and Cournot-Nash competitive equilibria under asymmetric takedown information is beyond the scope of this research. Borrowers' self-selection as a screening and risk-sharing device with optimal fee mix is also examined in Shockley and Thakor (1997), and reviewed in Greenbaum and Thakor (1995).

[4] In a well-known American variant, the upfront fee is charged in conjunction with a fee for either the amount actually borrowed and/or the unused portion of the commitment. In this case, the borrower who opts for a spot loan is charged the latter administrative cost in addition to the fee on the commitment unused balance.

[5] According to Shockley (1995) for the years 1989 and 1990, the mean upfront fee on corporate credit commitments was 27.4 basis points while the mean annual fee on commitment unused balances was 25.2 basis points. Angbazo *et al.* (1998) noted that these fees are declining since the mid-1990s due to stronger competition.

It now remains to explain why short-term loan commitments are considered as European put options within the BIS regulatory time frame. The aggregate value of all unfunded commitments with an initial term to maturity up to one year is reported as an off-balance sheet entry to the bank's annual consolidated statement. Yet, at the annual reporting date, the time remaining to commitment expiry is less than the initial one-year term for many outstanding commitments. So, in part (b) of the decision chart, the average time remaining to commitment maturity (T – s) has been standardized at 6 months with s denoting the valuation date. As Merton (1977) has argued for related loan guarantees, the length of time until the next audit date can also be interpreted as the remaining life of contract. At this point, the commitment can be exercised or not. If exercised, line funding results in an on-balance-sheet corporate loan and for the duration of the loan, the bank then holds a "vulnerable" repayment call on the borrower's assets since the latter may default on loan principal and/or loan interests[6]. Alternatively, the line is not drawn down and the commitment simply expires at date T (partial line takedown is dealt with in subsection 2.2.3 of the analytical model). Our analysis thus focuses on valuing the components "**split fees + commitment put**" at date s. For the sake of continuity, we shall refer to the problem statement in the rest of the paper.

2.2 Analytical valuation

The valuation of fixed-markup commitments is performed in three stages: discount bond, put value, and exposure to commitment credit risk. Our starting point is a two-factor option-valuation model along the lines of those examined by Merton (1973a), Rabinovith (1989) or Rebonato (1996).

2.2.1 The discount factor

Suppose that the risk-neutral diffusion process of the instantaneous spot interest rate, r(t), is

[6] Markup risk should not be confused with the risk of default by counterparties to off-balance sheet transactions such as swaps (see, among others, Duffee [1996] or Hull [2000]). In the latter case, this settlement risk is very similar to the one faced by the bank ***after*** the commitment has been exercised and the credit line drawn down: it holds a vulnerable counterparty call as the borrower

given by

$$dr = (\theta(t) - ar)dt + \sigma dz_r(t), \quad (1)$$

where a and σ are positive constants and $dz_r(t)$ is the differential of the standardized Wiener process $\{z_r(t): 0 \leq t \leq T\}$. At time t, r is reverting to the expected value $\theta(t)/a$ at the rate a, and $\theta(t)$ is chosen to ensure that the model fits the initial (that is valuation time s) term structure of interest rates. Suppose next that: (i) a default-free discount bond D pays one dollar on the date the credit line commitment expires, and (ii) this bond value is only a function of r(s) and the time to maturity, τ = T - s, namely D = D(r(s), τ). According to Hull and White (1990), the risk-neutral solution for D(r(s), τ) is:

$$D(s, T) = A(s, T)e^{-B(s, T)r(s)}, \quad (2)$$

where

$$B(s, T) = (1/a)[1 - e^{-a(T-s)}] \quad (3)$$

and

$$\ln A(s, T) = \ln[D(t, T)/D(t, s)] - B(s, T)[\partial \ln D(t, s)/\partial s] - (\sigma^2/4a^3)(e^{-aT} - e^{-as})^2(e^{2as} - 1). \quad (4)$$

Eqs. (2) to (4) define the price of the zero-coupon bond at valuation date s in terms of the prices of discount bonds at date t = 0. To compute eq. (2), we also need a term structure formula, the like $r(s) = 0.09 - 0.05e^{-0.18s}$. For this ascending yield curve, the instantaneous short rate at valuation date s is 4% p.a. and 4.43% p.a. at commitment expiry six months later. In addition, the variance of D(s, T) at date s, $\sigma^2_D(s) = [\sigma B(s, T)]^2$ is a known function of time to maturity, τ = T - s; this feature plays an important role in the put specification to be derived at the next stage.

2.2.2 Commitment put value

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may default on loan principal and/or interests.

Thakor *et al.* (1981) were the first to define the marked-to-market value of a credit line, an economic value often referred to as the **indebtedness value**, x. With regard to problem statement 2.1, the indebtedness value at date T is computed as

$$x_T = L\exp\{(\overline{m}_0 - m_T)(T_1 - T)\}, \tag{5}$$

where L is the line contractual value, $(T_1 - T)$ is loan duration once the commitment has been exercised and $(\overline{m}_0 - m_T)$ is the difference between $\overline{m}_0$, the date-0 fixed forward markup, and $m_T = (l_T - c_T)$, the date-T stochastic spot markup defined as the difference between the prime rate in the spot credit market, l_T, and the funding rate in the CD market, c_T. At date T, the commitment holder decides to draw on the line only if *ceteris paribus*[7] $\overline{m}_0 < m_T$, namely when the initial markup is less than the stochastic spot markup computed from primary credit and funding rates. For instance, when our illustrative 1.5% forward markup is combines with, say, a 2.5% spot markup, the markup differential in eq. (5) is negative at -1%. It follows that the inequality $x_T < L$ gives rise to the commitment put option as the indebtedness value is less than the line contractual value. The law of motion for the indebtedness value defined in eq. (5) is[8]

$$dx = x[\mu_x dt + \sigma_x dz_x(t)], \tag{6}$$

where μ_x and σ_x^2 are the instantaneous drift and instantaneous variance of the indebtedness lognormal

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[7] To maintain the neutrality of the trade-off between spot loan and credit under a commitment, the usage fee due at the exercise date, f^e_T, is assumed to match the administrative cost, c^a_T, that the borrower will pay for a spot loan. Otherwise, the markup differential $(\overline{m}_0 - m_T)$ becomes $(\overline{m}_0 + f^e_T - m_T - c^a_T)$ and eq. (5) is adjusted accordingly.

[8] The analytical development leading to eq. (6) can be found in Chateau and Dufresne (1999).

distribution[9] and $dz_x(t)$ the differential of the standardized Wiener process $\{z_x(t): 0 \leq t \leq T\}$. Once the indebtedness value and its dynamic process have been defined, we determine the terminal condition of the borrower's commitment put payoff:

$$P(x, T; L) = \max [\, 0, L - x_T \,] \qquad (7)$$

To derive the closed-form solution for P(x, T; L) at valuation date s, we substitute the discount bond value in Merton's [1973a, eq. (38)] put option formula. Following some manipulations (see Chateau and Dufrene [1999])[10], we obtain the value of the European commitment put when the default-free, short-term interest rate is stochastic as:

$$P(x, D(s, T), \tau, V; L) = D(s, T)LN(-d_-) - xN(-d_+), \qquad (8)$$

where

$$d_\pm = \{\ln[x/L.D(s, T)]/\sqrt{V}\} \pm \tfrac{1}{2}\sqrt{V},$$
$$V = \sigma^2_x\tau + (\sigma/a)^2[\tau - 2B(s, T)] + (\sigma^2/2a^3)(1 - e^{-2a\tau}) - 2(\rho\sigma\sigma_x/a)[\tau - B(s, T)], \qquad (9)$$

and where N(d) is the cumulative standard normal distribution at d, and x/(L.D(s, T)) the ratio of the indebtedness value to the line par value, the latter being expressed in exercise-value dollars payable at the commitment expiry date. The commitment put in eq. (8) is in the form of the corresponding Black and Scholes formula, $P_{B\text{-}S}$. There are two relevant differences, however. First, our stochastic, default-free short rate $r \equiv -[\ln(D(s, T)/A(s, T))]/B(s, T)$ differs from the risk-free short rate used in the B-S formula, $r_F \equiv -\ln(D(s, T))/\tau$. Second, the effect of the short-rate of eq. (1) on the put value is

9 From the evidence reported in Chateau (1995) and Chateau and To (1999), markup differentials are approximately normally distributed and indebtedness values approximately log-normally distributed.

10 Although the indebtedness value is not likely to trade directly, the difficulty is overcome (i) by appealing to Merton's (1973b) intertemporal CAPM as advocated by Thakor *et al.* (1981) or (ii) by observing as in Chateau and To (1999) that spot markups and indebtedness values constitute **quasi-prices** as they result from actual (equilibrium) prices in continuous primary lending and funding

captured through the last three terms on the right-hand-side of the instantaneous put variance, V in eq. (9). These capture the impact on P(x, D(s, T), τ, V; L) of: (i) σ, the short-rate volatility; and (ii) ρ, the correlation between changes in the short-rate and changes in the indebtedness value. It is also worth pointing out that the commitment put value corresponds to the "notional" discount of an off-balance-sheet contract.

2.2.3 Exposure to commitment credit risk

The commitment put is now combined with the fees collected by the bank to determine the commitment net value at date s, CNV_s. The upfront fee, $f^u{}_0$, guarantees the credit availability to the borrower; his subsequent choice between spot loan and credit under a commitment depends *ceteris paribus* on the risk trade-off between stochastic spot markup and fixed forward markup. This state-contingent decision is captured by an exercise indicator, $I = 1\{x_T < L\}$, that is equal to one if exercise occurs, and zero otherwise. Analytically, the following expression determines the commitment net value at date s

$$CNV_1 = f^u{}_0 \exp(r_F\tau_1) + I[\, f^e{}_T \exp(-r\tau_2) - P(x, D(s, T), \tau, V; L)\,] \quad \text{when there is exercise} \qquad (10a)$$
$$CNV_2 = f^u{}_0 \exp(r_F\tau_1) \quad \text{in the absence of exercise} \qquad (10b)$$

where the upfront commitment fee, $f^u{}_0$, is compounded at the risk-free rate $r_F \equiv -\ln(D(\tau_1))/\tau_1$, with $\tau_1 = s - t$. The usage fee being exercise contingent, $f^e{}_T$ in eq. (10a) is discounted at the default-free stochastic rate $r \equiv -[\ln(D(\tau_2)/A(\tau_2))]/B(\tau_2)$, with $\tau_2 = T - s$. In both cases, the interest rates are consistent with the price of the zero-coupon bond derived in subsection 2.2.1.

Recall, at this juncture, that the credit unit in problem statement 2.1 was standardized at \$100. Suppose at a given audit date, that: (i) the bank had previously written ten identical prime-rate credit commitments of \$100 each for a total of \$1,000, (ii) 60% of the commitments were already

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markets.

exercised[11], and (iii) for those exercised, the average takedown is presently 83.33% of each line maximum amount of \$100. So, from the bank viewpoint at the audit date, \$500 out of the potential \$1,000 are drawn down, that is a proportion of 50%. We propose to account for the exercise-cum-takedown feature by introducing the following bank-level simplification: fully drawn standardized credit units are used to cover the takedown proportion of the dollar total of aggregate commitments at a given date, the complementary fraction being the dollar aggregate of all unexercised and thus undrawn commitments. Then, p, the conditional average proportion, is

$$p = E[\, d.I \,|I = 1] = E[\, d \,|I = 1] = E[\, d \mid x_T < L], \qquad (11)$$

where E is the mathematical expectation and the exercise indicator I is combined with the takedown parameter, d. When there is full takedown of the \$100 credit unit, d = 1. In the absence of exercise and thus takedown, the complementary proportion is $(1 - p) = E[1 - d \mid x_T < L]$. For the sake of simplicity, we have selected a fixed proportion, p, and have reallocated partial takedown, $0 < d < 1$, to the two other proportions. Granted the empirical evidence reported in Morgan (1993), we have retained a

proportion p = 0.5: 50% of the dollar total of all commitments is drawn down and the other 50% is left unexercised. Combining these proportions to the CNVs from eqs. (10a) and (10b) allow us to determine the credit risk that the bank incurs when offering \$100 of credit under a commitment: namely

$$\text{Exposure} = p\ \{\ f^u_0 \exp(r_F\tau_1) + I\ [f^e_T \exp(-r\tau_2) - P(x, D(s, T), \tau, V; L)]\ \} + (1 - p)\ \{\ f^u_0 \exp(r_F\tau_1)\ \}. \qquad (12)$$

Equations (1) to (12) form the valuation programme of off-balance-sheet, short-term CL

[11] Morgan (1993) indicates that between 1988 and 1990, the fraction of the loan limit actually borrowed by prime-rate borrowers is about 55%; unfortunately, he is not reporting the number of commitments left unexercised.

commitments in the presence of two correlated factors, x and r. It is estimated in the next section.

3. SIMULATION RESULTS

3.1. Simulation

As embedded credit-risk derivatives, commitment puts are but notional values. We thus rely on simulations to compute their values, and our simulation parameters are based on the empirical evidence presented in Chateau and To (1999). In Tables 1 and 2, the indebtedness value is set at x = \$100, \$99.5, \$99, \$98.5 and \$98, respectively. For a line par value of \$100, the slightly in-the-money indebtedness values simulate small increases in the spot markup of the class of prime-rate borrowers over the yearlong commitment period. Granted these indebtedness values, simulation experiments are performed for a whole range of (x - r) correlation and short-rate volatility values. The value of ρ ranges over the value domain [-1.0 to 1.0] in Table 1; and the short-rate volatility, σ, varies over the empirically-relevant interval [2% p.a. to 10% p.a.] in Table 2. Four parameters are common to both simulations: a = 0.5 implies that 50% of the rate differential in the short-rate drift is adjusted each period, the ascending term structure, $r(s) = 0.09 - 0.05e^{-0.18s}$, is consistent with the 4.5% CD rate introduced in problem statement 2.1, $\sigma_x = 0.07$ indicates that the indebtedness-value volatility is a realistic 7% p.a., and $\tau = 0.5$ is the average time to commitment expiry. Finally, to compare eq. (8) to the B-S formulation, the latter commitment put, $P_{B\text{-}S}$, is computed with an adjusted discount factor $r_F \equiv -\ln(D(\tau))/\tau$.

Before reporting on the simulations, we first clarify the meaning of computed put values. Consider the very plausible scenario (represented by the bold-faced entries 9 to 12 in column (5) of Table 1) in which the indebtedness value x is slightly in the money at \$99 and the correlation between the indebtedness value and the short rate is mildly positive, $\rho = 0.2$ (the actual value for Canada over the period 1966 to 1999 is 0.289). According to entry 9 column (5), the estimate P = 1.38 means that the commitment put has an equilibrium value of 1.38% of the line par value if: (i) our prime-rate commitment with a 1.5%-p.a. fixed forward markup is priced when the stochastic spot markup is 2.5% p.a.; and (ii) the time remaining to commitment expiry is 6 months. The B-S one-factor estimate computed with an adjusted discount factor is $P_{B\text{-}S}$ = 1.40. Yet, despite this normalization, the B-S put value remains slightly higher than the two-factor put value: according to entry 10, column (5), the latter is under-priced by 2.0% in terms of the B-S put value. The put estimate is next used in entry 11 of column (5) to compute CNV_1, the commitment net value when the line is completely drawn down: the negative CNV_1 corresponds to a 88¢ notional discount per $100 of nominal credit. However, if the commitment is left unexercised, CNV_2 constitutes a premium of 25.6 cents per $100 of credit; as noted in footnote (a) of Table 1, CNV_2 is the same value in all cases. Finally, with a conditional exercise-cum-takedown proportion of 50%, the bank's risk-weighted exposure to commitment credit risk, shown in entry 12 of column (5), turns out to be a notional liability of 31 cents per $100 of credit offered.

3.2. Commitment put values, CNVs and the bank's credit-risk exposure

Two revealing tendencies of commitment put values are emerging from Tables 1 and 2 below. Table 1 illustrates the first tendency that value differences between P and $P_{B\text{-}S}$ are due to the (x - r) correlation. In the absence of correlation between indebtedness value and short rate, the two-factor put values, P in

TABLE 1

Values at valuation date-s of: i) P, the European commitment put of the two-factor model, and $P_{B\text{-}S}$, the B-S single-factor put value with an adjusted discount factor; ii) Bias (in %) = $[P - P_{B\text{-}S}]/P_{B\text{-}S}$; iii) commitment net values: CNV_1 when the commitment is fully exercised and CNV_2 when it is left unexercised[a]; and iv) Exp. = exposure to commitment credit risk. Parameter definition: a = reversion parameter of the short-rate drift; L = credit line nominal value in $; r = short-term rate of interest, in % p. a.; rho = indebtedness value-short rate correlation; σ = volatility of the short-term interest rate, in % p.a.; σ_x = indebtedness-value volatility, in % p.a.; τ = time to commitment expiry, in years; x = indebtedness value in $.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
#	ρ	-1	-0.5	-0.2	0	0.2	0.5	1.0
1 x = $ 100	P	1.29	1.18	1.12	1.07	1.02	0.95	0.83
2 $P_{B\text{-}S}$ = 1.05	B %	22.8	12.6	6.2	1.9	-2.5	- 9.4	-21.3
3	CNV_1	-0.79	-0.68	-0.61	-.57	-0.52	-0.45	-0.33

4	Exp.	- 0.27	-0.21	-0.18	-.16	-0.13	-0.10	-0.04
5 x = \$ 99.5	P	1.47	1.36	1.29	1.24	1.19	1.12	0.98
6 $P_{B\text{-}S}$ = 1.22	B %	20.3	11.2	5.6	1.7	-2.3	- 8.4	-19.2
7	CNV_1	-0.97	-0.86	-0.79	-0.74	-0.69	-0.62	-0.48
8	Exp.	-0.36	-0.30	-0.27	-0.24	-0.22	-0.18	-0.11
9 x = \$ 99	P	1.66	1.55	1.47	1.43	**1.38**	1.30	1.16
10 $P_{B\text{-}S}$ = 1.40	B %	18.0	10.0	5.0	1.5	**-2.0**	-7.5	-17.2
11	CNV_1	-1.16	-1.05	-0.98	-0.93	**-0.88**	-0.80	-0.66
12	Exp.	-0.45	-0.40	-0.36	-0.34	**-0.31**	-0.27	-0.20
13 x = \$ 98.5	P	1.87	1.75	1.68	1.64	1.58	1.50	1.37
14 $P_{B\text{-}S}$ = 1.61	B %	16.0	8.9	4.4	1.3	-1.8	-6.7	-15.3
15	CNV_1	-1.37	-1.26	-1.18	-1.13	-1.08	-1.00	-0.87
16	Exp.	-0.56	-0.50	-0.46	-0.44	-0.41	-0.37	-0.31
17 x = \$ 98	P	2.10	1.98	1.91	1.86	1.81	1.73	1.59
18 $P_{B\text{-}S}$ = 1.84	B %	14.1	7.8	3.9	1.2	-1.6	-5.9	-13.5
19	CNV_1	-1.60	-1.48	-1.41	-1.36	-1.31	-1.23	-1.09
20	Exp.	-0.67	-0.61	-0.58	-0.55	-0.53	-0.49	-0.42

Common parameter values: a = 0.5; L = 100; $r(s) = 0.09 - 0.05e^{-0.18\,s}$; $\sigma = 0.04$; $\sigma_x = 0.07$; and $\tau = 0.5$.

[a] CNV_2 = (25¢)exp[(0.044303)(0.5)] constitutes a premium of 25.6¢ *in all cases*.

TABLE 2

Values at valuation date s of: i) P, the European commitment put of the two-factor model, and $P_{B\text{-}S}$, the B-S single-factor put value with an adjusted discount factor; ii) Bias (in %) = $[P - P_{B\text{-}S}]/P_{B\text{-}S}$; iii) commitment net values: CNV_1 when the commitment is fully exercised and CNV_2 when it is left unexercised[a]; and iv) Exp. = exposure to commitment credit risk. For definition of parameters, see Table 1.

		(1)	(2)	(3)	(4)	(5)
#	σ	0.02	0.04	0.06	0.08	0.10
1 x = \$ 100	P	1.03	1.023	1.025	1.037	1.058
2 $P_{B\text{-}S}$ = 1.05	B %	-1.7	-2.5	-2.4	-1.2	-0.8

3		CNV_1	-0.532	-0.524	-0.525	-0.537	-0.559
4		Exp.	-0.138	-0.134	-0.135	-0.141	-0.152
5	x = $ 99.5	P	1.20	1.19	1.19	1.20	1.23
6	P_{B-S} = 1.22	B %	-1.6	-2.3	-2.1	-1.1	0.7
7		CNV_1	-0.70	-0.691	-0.693	-0.705	-0.728
8		Exp.	-0.222	-0.218	-0.219	-0.225	-0.236
9	x = $ 99	P	1.386	**1.376**	1.379	1.391	1.415
10	P_{B-S} = 1.40	B %	-1.4	**-2.0**	-1.9	-0.1	-0.7
11		CNV_1	-0.886	**-0.877**	-0.878	-0.892	-0.916
12		Exp.	-0.315	**-0.31**	-0.312	-0.318	-0.33
13	x = $ 98,5	P	1.592	1.583	1.585	1.598	1.622
14	P_{B-S} = 1.61	B %	-1.2	-1.8	-1.7	-0.9	0.6
15		CNV_1	-1.093	-1.084	-1.086	-1.10	-1.12
16		Exp.	-0.419	-0.414	-0.415	-0.421	-0.433
17	x = $ 98	P	1.819	1.81	1.812	1.825	1.849
18	P_{B-S} = 1.84	B %	-1.1	-1.6	-1.5	-0.8	0.5
19		CNV_1	-1.32	-1.31	-1.312	-1.326	-1.349
20		Exp.	-0.532	-0.527	-0.528	-0.535	-0.547

Common parameter values: a = 0.5; L = 100; $r(s) = 0.09 - 0.05e^{-0.18\,s}$; $\rho = 0.2$; $\sigma_x = 0.07$; and $\tau = 0.5$.

[a] $CNV_2 = (25¢)\exp[(0.044303)(0.5)]$ constitutes a premium of 25.6¢ *in all cases*.

column (4) of Table 1, are slightly higher than the corresponding values of the normalized put, P_{B-S}. According to the bias estimate (B %), the overvaluation ranges from [1.9% to 1.2%] for indebtedness values moving progressively deeper in the money. For nonzero (x - r) correlation in contrast, the degree of over- or under-valuation of P with regard to P_{B-S} values can be substantial. The magnitude of value divergences is however decreasing as indebtedness values move deeper in the money: i.e., from a value range of [-21.3% to 22.8%] for an even indebtedness value, x = $100, to a narrower range of [-13.5% to 14.1%] for the deep-in-the-money value, x = $98. The second pattern that emerges from Table 2 concerns the effect of the short-rate volatility on the commitment put values. Visual inspection of the table reveals that percentagewise the P biases are rather small for all volatility values. Even if the short-rate volatility doubles or even triples in the empirically-relevant range [2% p.a. to 10% p.a.], the degree of systematic under- as well as over-estimation of P values with regard to the $P_{B\text{-}S}$ estimates ranges from -2.5% to 0.8%. A comparative inspection of both tables also reveals that the magnitude of put biases is significantly greater for the (x – r) correlation than for the short-rate volatility.

This evidence then raises the question: which model of P or $P_{B\text{-}S}$ should be used in computing the commitment risk-adjusted balance that enters the calculation of the bank's solvency ratio? Even after adjusting its discount factor, the single-variable model produces put over- as well as under-valuation with regard to the two-factor model. Thus normalizing the B-S commitment put formula is not sufficient, since systematic biases are remaining due to either the (x - r) correlation or the short-rate volatility. In contrast, the two-factor model is richer and its more realistic commitment put estimates may be better suited to the capital adequacy problem examined later on in Section 4.

In a chain reaction, commitment put values affect the CNVs and the bank's credit-risk exposure. In this regard, scenario x = $99 corresponding to entries 11 and 12 column (5) in Table 1 is again representative: when the line is exercised and fully drawn down, CNV_1 constitutes a **net notional discount** of 88¢ per $100 of credit provided. More concretely, if the bank were to carry off-balance sheet a 1-billion tranche of short-term unused commitments, it ought to simultaneously report a notional liability of 8.8 million. If the commitment is left unexercised as in footnote (a) of Table 1, the bank collects a 25.6¢ premium per $100; and when a 50% exercise-cum-takedown proportion is assumed, the bank's exposure is approximately 31¢ per $100 of credit offered as per entry 12 in column (5). Or, to put it differently, a $1-billion tranche of short-term commitments carries with it an off-balance-sheet **credit-risk liability** of $3.1 million. Irrespective of the (x - r) correlation or volatility parameter chosen in Tables 1 and 2, the credit-risk exposure of short-term commitments constitutes a net liability for all indebtedness values. Under rational pricing, the bank should then strive to achieve at least a break-even exposure. To wipe out any remaining (negative) exposure, it could then decide to: 1) rise its future fees by altering the combination of upfront and rear-end fees, 2) increase the fixed forward markup of the class of prime-rate borrowers, or 3) resort to a combination of both.

In the last section, the above simulations are now used to determine the effect of bank commitments on the risk-based capital requirement.

4. LINKING COMMITMENT CREDIT RISK TO CAPITAL SUFFICIENCY

The Basle capital rules deal with ***credit risk*** (see BIS [1988] or Santos [2001])[12]: they require that standard risk-adjusted balances be determined for each off- as well as on-balance-sheet instrument and their aggregate value be weighted against a definition of regulatory capital. To calculate risk-adjusted values, off-balance-sheet contractual amounts are initially converted by way of **credit conversion factors** to "credit equivalent [on-balance sheet] amounts"; which in turn are weighted by appropriate **principal risk factors** to determine "risk-adjusted balances". Since the end of 1992, a minimum total capital requirement of 8% applies to such balances. In the Amendments to the Accord (see BIS 1999a, b), however, the commitment original term to maturity (the less-than or over-one-year dichotomy) is not recognized anymore as an acceptable criterion. Yet the BIS still awaits the industry proposals regarding these off-balance sheet instruments.[13] To illustrate our subject, data regarding commitments and loans are presented in Table 3 for a large international bank, the Royal Bank of Canada as at October 31, 2000.

TABLE 3: BIS accounting-based valuation of credit risk: from off-balance sheet commitments to on-balance sheet loans.

	Off-balance sheet commitments		On-balance sheet loans
With an original term to maturity	< 1 yr	> 1 yr	

[12] Since the beginning of 1998, a separate guideline regarding market risk (see BIS 1995) is also in force; in this case, the banks are allowed to use their own VaR models to compute the amount of regulatory capital required for market risk.

[13] An analysis and critique of the BIS new guidelines can be found in Krahnen and Weber (2001), Hammes and Shapiro (2001), or Wahrenburg and Niethen (2000).

(1) Contractual amount, $ in billions	98.0	41.6	n.a.[1]
(2) **Credit conversion** factor, in %	0	50%	n.a.[1]
(3) Credit-equivalent amount, $ in billions	nil	20.8	108.7
(4) **Principal risk** factor, in %	0	100%	100%
(5) BIS risk-adjusted balance, $ in billions	nil	18.9	89.5

[1] n.a. = not applicable
Source: The Royal Bank of Canada annual report as at October 31, 2000.

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Consider now how commitments to extend credit with an original term to maturity up to 1 year are treated under the BIS present guideline. According to line (3) of Table 3, their credit equivalent amount is nil since the credit-conversion factor applied to the commitment contractual amount ($98.0 billion on line (1)) is 0%; and their risk-adjusted balance is accordingly also nil as the risk factor for such commitments is also 0%. ***There is thus no link between actual or potential risk and the capital charge as short-term commitments do not affect the risk-adjusted capital requirement at any point in time or on a continuous basis***. The same is not true however for longer-term commitments and on-balance sheet loans. On line (5), the risk-adjusted balance of over-one-year commitments is $18.9 billion and that of other (mainly corporate) loans is $89.5 billion, and, in both cases, the principal risk weight is 100% according to line (4). On line (1) also, the contractual amount of short-term commitments ($98.0 billion) is larger than that of longer-term commitments ($41.6 billion) and sizeable at any rate with regard to on-balance-sheet corporate loans, $108.7 billion shown on line (3). More concretely, the balance-sheet amount of outstanding loans is $108.7 billion while the total amount of off-balance-sheet unused commitments is $139.6 billion, with $98.0 billion or 70.2% of them being riskless according to the BIS **accounting-based** valuation of credit risk.

Structural changes in the relationships between these three aggregates have however taken place since the introduction of the BIS capital guideline in 1989. This is illustrated below, for the

Royal Bank of Canada again. The structural shift away from longer-term (> 1 year) commitments has been

Contractual amount (in $ billions) of unused commitments with an initial term:

		1989	1991	1993	1995	1997	1999	2000
≤ 1 year	$	40.9	34.9	44.5	47.5	68.7	83.4	98.0
> 1 year	$	28.8	23.1	23.7	34.7	35.6	45.8	41.6
Corporate loans	$	55.0	63.0	59.2	64.5	80.2	100.8	108.7

quite significant. The amount of less-than-one-year commitments has increased by 139.6% over the eleven-year period, rising from $40.9 billion in 1989 to $98.0 billion in 2000, and their share of the commitment total has grown significantly from 58.7% to 70.2%. Simultaneously, the 44.4% rate of growth in longer-term commitments over the same period has resulted in their share of the commitment total shrinking from 41.3% to 29.8%. The reason of the shift is simple: banks adjust their commitment portfolio toward those in the low weight class and away from those in the high weight class.[14] By way of contrast, the volume of corporate loans has increased by 97.6% in eleven years, rising from 55 billion to 108.7 billion. This growth rate being however lower than that of short-term commitments, the ratio of short-term off-balance sheet commitments to on-balance sheet corporate loans has increased from 74.4% to 90.1% over the same period. These observations have a direct bearing on the link between actual incurred risk and the capital charge, as $1 of outstanding corporate loan attracts an 8% ratio whereas the same dollar of short-term unexercised commitment requires no capital (0% capital sufficiency). Yet, the moment short-term unused commitments are exercised, they become problematic in two respects. First, they become "credit risk" expensive since among the off-balance-sheet instruments, they initially had the lowest capital requirement at 0%. Second, this asset substitution may crowd out other less (credit) risky and so less capital-expensive

[14] This regulatory arbitrage is also recognized in André *et al.* (2001).

off- as well as on-balance-sheet instruments (for instance, there is no capital requirement for short-maturity government securities). It may simultaneously trigger a funding crisis: absent sufficient core deposits, the bank may decide to fund its exercised commitments with more expensive CDs or may even be forced to sell some illiquid securities. Contrariwise, if the intermediary attempts to curtail commitment offering, this course of action will reduce its fee income and accordingly, its current as well as future profitability. The present guideline has thus given rise to some sort of "ideal commitment cycle" from the bank's point of view. It can be formulated as

PROPOSITION I: ***(1) book a large volume of short-term commitments of which statistically only a fraction (say 50%) will be funded; (2) "securitize" next the resulting loans so as to move them off-balance sheet; and (3) repeat the procedure as rapidly as possible so as to optimize fee income and simultaneously minimize the bank's credit-risk capital requirement***.

At this juncture, we are in a position to offer a market-based alternative to the BIS accounting-based approach: simply combine the numerical values obtained in Table 1 with the data presented in Table 3. In short our exercise-cum-takedown proportion plays the role of the BIS conversion factor and our commitment put value that of the BIS principal risk factor. This approach, which is similar to that applied to the other off-balance sheet derivative instruments, is illustrated numerically in Table 4. Consider the contractual amount (L = \$98.0 billion) of short-term unused commitments reported earlier in Table 3 in conjunction with the benchmark scenario of Table 1, in which x = \$99 captures a mild decline in the creditworthiness of prime-rate borrowers. With an exercise-cum-takedown proportion of 50%, the contractual amount of off-balance-sheet short-term commitments is converted to an on-balance-sheet credit-equivalent amount of

$$\$98.0 \text{ billion} \times (0.5) = \$49.0 \text{ billion}$$

that is shown on line (3) of Table 4. Recall also that the value of any European put can be decomposed

TABLE 4: Fair value or derivative-based risk valuation of off-balance sheet commitments with an original term to maturity less than one year.

Concepts	Illustration
(1) Contractual amount, $ in billions	98.0
(2) **Exercise-cum-takedown proportion**, in percentage	50%
(3) Credit-equivalent amount, $ in millions	49.0
(4) **Commitment put value of two-factor model**: entry 9 in column (5) of Table 1	0.0138 per billion
(5) **Option-based** risk-adjusted balance, $ in millions	676.2

into intrinsic and time components. So, here, the intrinsic value (L - x) of the commitment put (i.e., today's credit exposure) plus its time component, if any, (corresponding to the potential future credit exposure) is equal to the European commitment put value (namely the cost of contractual risk). Since this contractual risk captures the "principal risk factor", the risk-adjusted balance of short-term commitments shown on line (5) is:

$49.0 billion × 0.0138 (= commitment put value per $ **billion**) = $676.2 million.

This risk-weighted balance is substantially different from zero, the BIS accounting-based balance; short-term commitments thus do attract a positive capital charge. This option-based approach can be formalized[15] in

[15] While not reported in Table 4, the same procedure (p×L×P) also applies to the commitments with a term longer than one year reported in Table 3. Our approach thus also remedies the

PROPOSITION II: (1) ***Compute the indebtedness value of short-term unused credit commitments, x, and subtract it from the line par value, L; (2) add this current commitment exposure (L - x) to the potential future exposure captured by the time component, if any, of the two-factor commitment put value, P; (3) compute next the credit-equivalent amount by applying the exercise-cum-takedown proportion, p, to the contractual amount of short-term commitments; and (4) weight this credit equivalent amount by the European put, P from (2), to arrive at the risk-adjusted balance of this off-balance sheet instrument.***

5. SUMMARY

This research offers to remedy the idiosyncratic situation in which the BIS accounting-based standards are not an adequate measure of the banks' credit risk of off-balance-sheet short-term credit commitments. To restore the link between commitment actual and potential risk and the capital charge, we first value analytically the commitment put in a two-factor model in which the default-free short-term interest rate is stochastic. Compared to the (single-factor) Black-Scholes put formula, the proposed model highlights the role played in commitment pricing by (i) the volatility of the short-term interest rate, and (ii) the correlation between the unanticipated changes in the short rate and those in the indebtedness value. Once computed, the put value is combined with the credit-line fees and a conditional exercise-cum-takedown proportion to determine the commitment net value and ultimately, the bank exposure to commitment credit risk. The main pattern that emerges from the simulation experiments is: over- or under-valuations of the two-factor commitment put model with respect to the Black-Scholes formula are greater when the (x – r) correlation is varying in the range [-1 to 1] than when the short-rate volatility increases from 2% p.a. to 10 % p.a. This observation makes it thus natural to use the more versatile two-factor model to link commitment credit risk to the bank's regulatory capital requirement. The main policy implications of this link are

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present artificial dichotomy in commitment maturity.

that (i) the bank's exposure to commitment credit risk gives rise to a notional liability, and hence, (ii) the positive risk-weighted balance of short-term commitments should attract a capital charge. In doing so, the procedure proposes to replace the accounting-based conversion and principal-risk factors and the commitment maturity dichotomy by the market-based concepts of commitment put value and exercise-cum-takedown that apply to all commitments without distinction of initial term to maturity.

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